Magnetic diffusion, inductive shielding, and the Laplace transform (Supplemental Material)

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In preparing this manuscript, we compiled an extensive – but certainly not exhaustive – catalogue of recent pedagogical papers that are related to the interaction of magnetic fields and conducting materials. We have attempted to group them here in a list of references by common theme: forces on moving magnets and MAGLEV;[1–4] magnetic braking;[5–15] permanent magnets sliding down conductive planes[16–19] or falling through conductive tubes;[20–34] eddy currents, skin depth, and inductive shielding;[35–48] and Thomson's jumping ring.[49–54] There is significant overlap in the content of these and other related papers,[55–60] of course, and such groupings cannot be considered exclusive. Still it is hoped this compilation will provide a convenient and fairly complete overview of the pedagogical literature on these topics and will also function as a quick guide for physics educators seeking new ideas and insights. Other more research-focused papers that we found useful are also listed here.[61–64]

In addition to the references cited in the main article regarding time-varying fields and conducting tubes, there are several other works on this topic[65–70] that may be of interest to the reader, as well.

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